STRUCTURAL STABILITY OF FINANCIAL AND ACCOUNTING SIGNALING EQUILIBRIA

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ABSTRACT

Financial and accounting signaling equilibria are proven to be structurally stable under perturbations of their environments. This is true for each of these possible financial or accounting market signals: dividends, capital structure, retained entrepreneur insider positions, stock splits, share repurchase, the issue of new securities, the call of convertible bonds, tender offers, quality of auditor or investment banker, the choice between LIFO versus FIFO accounting methods, and the medium of exchange in acquisitions.
I. INTRODUCTION

Since the seminal work of Spence (1973, 1974) on job market signals, there have been many applications of signaling theory to finance and accounting, suggesting that corporate financial or accounting decisions by insider managers can credibly communicate private information about future earnings to outside potential investors. Leland and Pyle (1977) demonstrate that the amount of stock held by an entrepreneur could signal the quality of her or his risky project. Numerous models analyze the possible informational content of dividends. Dissipative dividend signaling models, such as Miller and Rock (1985) and John and Williams (1985), appeal to Riley’s (1979) assumptions A1–A6 for the existence of an informational equilibrium. This entire literature has been summarized critically by Miller (1987). These approaches to resolving the so-called dividend puzzle as to why firms make positive dividend payments are shown in this paper to be structurally stable, in a precise sense, which ensures that chaotic dividend behavior does not occur.

The principal result is that the Pareto-undominated signaling equilibrium is a continuous function of the underlying environment for any financial or accounting signaling model satisfying Riley’s (1979) assumptions A1–A6. This is desirable for at least these reasons:

1. A concern on the part of both the academic financial community and practitioners as to what exactly the objective function of the firm is or should be under asymmetric information between the managers inside the firm and potential investors outside the firm. The traditional answer to such a question under the condition of symmetric information (namely, the appropriate goal of the firm is to maximize the wealth of its current stockholders) is liable to several interpretations when managers have private information. This lack of consensus as to what firms maximize led Miller and Rock (1985) to postulate the objective of the firm to be maximization of a “weighted social welfare function” that balances the conflicting interests of the shareholders who plan to sell their shares next period versus those who do not.

2. Recent work proves the stability, in the game-theoretic sense, of market signaling outcomes. The Pareto-dominant and zero-profit separating equilibrium has been shown by Riley (1985) to be the Nash equilibrium of a multiple-principal, multiple-agent game, by Engers and Fernandez (1987) to be among the perfect Nash equilibria of a specific extensive-form game, and by Cho and Kreps (1987) to be the sequential equilibrium that satisfies their particular intuitive criterion on beliefs. Since signaling phenomena have been shown to be stable in one sense, namely, that of Nash equilibria and its various refinements, one is led naturally to ask if they are stable in other senses. Huang (1985) has demonstrated the stability of Spencian equilibria in the asymptotic sense under Bayesian updating. The question of stability of signals in a structural sense is precisely what is addressed in this paper.

3. A motivation to study the structure of an equilibrium notion once it has been shown to be a nonvacuous concept. This type of research paradigm was pioneered by Debreu (1970) when he demonstrated that the competitive equilibria of a pure exchange economy are generically finite.

4. A desire to perform comparative statics, as is done in Appendix B of Miller and Rock (1985). They examine how the Pareto-undominated signaling equilibrium with a logarithmic production function varies as they vary a parameter. For that particular case, the Pareto-undominated equilibrium is clearly a differentiable (and hence continuous) function of their parameter as they analytically solve for the Pareto-undominated signaling equilibrium. This is not clear for other production functions not resulting in closed-form solutions. That such a sensitivity exercise can be performed more generally is demonstrated here.

5. A bounded rationality approach that recognizes agents might not be able analytically to handle the complexity of the relationships linking such exogenous parameters as firm objective functions and signaling cost functions to such endogenous variables as the stock prices of various firms. Due to the inherent computational intractability of equilibrium relationships, financial decision-makers often choose to approximate their world by using analytically manageable parametric specifications. Only approximately correct understanding of the underlying signaling environment might not lead to a signaling equilibrium or to a signaling equilibrium that is not even approximately close to the true signaling equilibrium. Structural stability is thus a natural minimal requirement of any model that involves asymmetric information. Hindy (1989) introduces a very similar criterion termed robustness in an equilibrium model of futures markets dynamics.

6. A prerequisite to conducting econometric or empirical testing of accounting or financial signaling theoretical models, due to possible measurement or specification errors. Balasko (1988, p. 95) justifies local use of linearized general equilibrium analysis by appealing to the smoothness of the natural projection from prices and initial endowments to initial endowments.

The rest of this paper is organized as follows. In Section II, the Miller and Rock (1985) dividend signaling model is summarized. The applicability of assumptions A1–A6 of Riley (1979) to Miller and Rock (1985) and John and Williams (1985) is discussed. In Section III, the structural stability of financial and accounting signaling equilibria is proven for any model that satisfies assumptions A1–A6 of Riley (1979). In Section IV, conclusions and extensions to multidimensional financial and accounting signaling models are considered.

II. A SIGNALING APPROACH TO DIVIDENDS

In order to be specific, we focus on Miller and Rock’s (1985) model. As has been pointed out, however, the method of reasoning used in Section III applies to all
Financial or accounting signaling models that satisfy assumptions A1–A6 of Riley (1979). Thus, for example, our main result holds for Stoughton’s (1988) explanation of the informational role of corporate merger and acquisition offers and Welch’s (1989) signaling analysis of the underpricing of initial public offerings.

The first six equations of Miller and Rock (1985) are:

First-period random earnings:

\[ X_1 = F(l_0) + \varepsilon_1 \]  

Second-period random earnings:

\[ X_2 = F(l_1) + \varepsilon_2 \]
\[ = F(X_1 + B_1 - D_1) + \varepsilon_2 \quad \text{(by the budget equation).} \]  

The cum-dividend firm value:

\[ V_1 = D_1 + (1 + \delta)^{-1}E[X_2] - B_1 \]
\[ = D_1 + (1 + \delta)^{-1}[F(l_1) + E(\varepsilon_2)] - B_1 \quad \text{[by Eq. (2)]} \]
\[ = D_1 + (1 + \delta)^{-1}[F(l_1) + \gamma \varepsilon_1] - B_1, \quad \text{as } E(\varepsilon_2 | \varepsilon_1) = \gamma \varepsilon_1 \]  

The firm’s budget constraint:

\[ X_1 + B_1 = D_1 + I_1 \]  

or “net cash flow” = “net dividends”:

\[ X_1 - I_1 = D_1 - B_1 \]  

The firm’s true value:

\[ V_1 = X_1 - I_1 + (1 + \delta)^{-1}[F(l_1) + \gamma \varepsilon_1] \]  

Henceforth, we use the variable \( D_1 \) to denote the “net” dividend, that is, the difference between the dividend and the level of borrowing. The above structural equations describe the investment possibilities for a smooth neoclassical production function technology \( F \) satisfying, for all feasible \( l, F(l) \geq 0, F'(l) > 0, F''(l) < 0; \text{ and } F(0) = 0 \). The firm’s random-earnings process \( X \) is assumed to have additive shock terms \( \varepsilon_1 \) and \( \varepsilon_2 \) satisfying \( E(\varepsilon_1) = E(\varepsilon_2) = 0 \) and \( E(\varepsilon_2 | \varepsilon_1) = \gamma \varepsilon_1 \), with \( \gamma \) being fixed and common knowledge. Informational asymmetry concerns the realized value of the random disturbance term \( \varepsilon_1 \):

\[ \Phi^d = \{X_1, I_1, d_1\} = \{l_0, \varepsilon_1, I_1, D_1\} \]  

\[ V_1^d = D_1 + (1 + \delta)^{-1}[F(l_1) + \gamma \varepsilon_1] \]  

Equations (7)–(10) represent both the information sets and corresponding perceived firm valuations for insider managers and the outside market investors, respectively. To reconcile the conflicting interests between those present stockholders who plan on selling their shares next period and those remaining stockholders who intend holding onto their shares for another period (the long run in such a two-period model as this), Miller and Rock (1985) use the weighted-average objective function:

\[ W = kV_1^p + (1 - k)V_1^d \]
\[ = D_1 + (1 + \delta)^{-1}[F(l_1) + \gamma \varepsilon_1 | \Phi^m] \]  

The solution to (11) is an optimal \( D \) given \( X \). Working in the other direction, for a fixed level of dividends \( D \) there are levels of current earnings \( X \) that make \( D \) optimal. This is a correspondence denoted by \( X(D) \). Whenever Miller and Rock’s (1985) model satisfies Riley’s (1979) assumptions A1–A6, this correspondence is nonempty and single-valued, so that \( X(D) \) is a bona fide function. In order for the market valuation to be rational this condition must hold:

\[ V^m(D) = V^d(X(D), D) = V^d(X, D) \]  

In Appendix A, Miller and Rock (1985) list six assumptions and appeal to Riley’s (1979) assumptions A1–A6 to conclude the existence of dividend signaling equilibria. But if outsiders are to forecast a nonnegative level of investment, \( I \geq 0 \), dividends must satisfy \( D \leq X(D) \). Thus, assumption A2 in Miller and Rock (1985) does not correspond to assumption A2 in Riley (1979), because the feasible domain of dividend signals cannot be specified independently of the signaling equilibrium price function \( X(D) \). Existence of an equilibrium might not be a problem necessarily, however, because Riley’s (1979) assumptions form a set of sufficient (as opposed to necessary) conditions for the existence of equilibrium.

We illustrate this problem with the production function of Appendix B in Miller and Rock (1985), namely,

\[ F(l) = a \ln(l + b), \quad \text{with } a, b > 0 \]
Note the equilibrium price function is
\[ X = (\alpha - 1)\beta^{-1} + D + \beta^{-1} \exp[-\beta(D - D^*)] \]
with \[ \alpha = [a - (1 + i)b]/ka \quad \beta = (1 + i)/ka \]
Thus, because \( X \in [X, \infty) \), as \( X \to \infty, D \to \infty \) and
\[ I = X - D = (\alpha - 1)\beta^{-1} + \beta^{-1} \exp[-\beta(D - D^*)] \to (\alpha - 1)\beta^{-1} \]
which is greater than or equal to zero if \( (\alpha - 1) > 0 \) as \( \beta^{-1} > 0 \). But,
\[ \alpha - 1 > 0 \iff [(a - (1 + i)b)/ka] > 0 \iff a(1 - k) > b(1 + i) \]
Hence, unless additional conditions are imposed on the permissible values of \( a, b, i, \) and \( k \), high-value firms will end up choosing negative levels of investment. For those values of \( a, b, i, \) and \( k \) with \( a(1 - k) > b(1 + i) \), the domain of \( X \) must have a finite upper bound, which depends on \( a, b, i, \) and \( k \).
Similarly, if outside investors are to have rational expectations in the John and Williams model (1985), the new capital raised, \( D \), must be less than or equal to \( X(D) \). Thus, \( D - I \leq X(D) - I \). Once again, the domain for dividend signaling decisions cannot be specified independently of the equilibrium price function \( X(D) \).
To be precise, the optimal dividend choice \( D(X) \) given by Eq. (13) in John and Williams (1985) is not feasible for some values of \( I \) and \( \tau \), the rate dividends are taxed at. Consider the case of both \( C \), the firm's existing cash holdings, and \( L \), the demand for liquidity by existing shareholders, being zero. John and Williams (1985) solve for the optimal dividend:
\[ D(X) = (I/\tau) \ln X, \quad \text{with } X = 1 \text{ and } D(X) = 0 \]
From the difference between the maximum feasible dividend, namely, \( X - I \), and the optimal dividend given by John and Williams,
\[ D(X) = X - I - (I/\tau) \ln X \]
This difference achieves its unique minimum at \( X = (I/\tau) \). Therefore, the minimized value of this difference is given by
\[ \min_X [X - I - D(X)] = (I/\tau) - (I/\tau) \ln[(I/\tau)] \]
Notice that
\[ (I/\tau) - (I/\tau) \ln[(I/\tau)] < 0 \iff I > \tau \exp(1 - \tau) \]
So for low values of \( X \), unless \( I < \tau \exp(1 - \tau) \), there will be infeasible values of \( D(X) \) given by Eq. (13) in John and Williams (1985). If \( I > \tau \exp(1 - \tau) \), there will be a feasible \( D(X) \) solution if the lower bound for \( X \) is greater than the pair of solutions to the equation \( X - I - D(X) = 0 \), which will be greater than one.

In addition, the John and Williams model (1985) does not satisfy assumption A3 of Riley (1979). This follows because the firm's budget constraint is \( D + I \), which is the new equity capital raised. As \( I \) was assumed to be observable, instead of viewing \( D \) as the signal, think of \( \alpha \), the fraction of the firm originally sold to market outsiders, as the signal. Then \( X(\alpha) \) is the market price of the firm and \( \alpha X(\alpha) \) is the capital raised, with the excess of that over \( I \) being dividend payments. If \( I = 0 \), then existing shareholders use their dividends to buy \( D \) dollars in stocks from outsiders. So insiders have a total fractional ownership of their firm equal to
\[ 1 - \alpha + D/I(X(\alpha)) = 1 - \alpha + [\alpha X(\alpha) - I]/X(X(\alpha)) = 1 - I/X(\alpha) \]
(13)
Since dividends are taxed at the rate \( \tau \), the purchase of \( D \) dollars in shares of the firm from outsiders requires \( \tau[\alpha X(\alpha) - I] \) to be invested by insiders. Thus, the objective function of insiders of the firm is
\[ \max_a U(X, \alpha, X(\alpha)) = -\tau[\alpha X(\alpha) - I] + [1 - X/X(\alpha)]^\gamma \quad \text{s.t. } \alpha \in [0,1] \]
(14)
Since \( \partial U/\partial X(\alpha) = -\tau X/X(\alpha)^2 \) could be positive or negative, \( U \) is not strictly increasing in \( X(\alpha) \).

III. STRUCTURAL STABILITY OF DIVIDEND SIGNALING

The objective function \( W(X, D, V^m) \), is the analog of the seller's utility function in Riley (1979). In order to have a notion of closeness between two members of the vector space \( S \) of possible firm objective functions, that set \( S \) is made into a metric space, with the distance function \( d(x, y) \) defined by the \( C^2 \) uniform-convergence norm of the difference \( x - y \). Under this norm, a function's "length" is the maximum value the sum of its values and those of its first and second partial derivatives takes on over its domain if that domain is compact. The resulting metric is the Whitney \( C^2 \) metric. Likewise, the set \( P \) of all possible dividend signaling functions \( D(X) \) is endowed with the \( C^1 \) uniform-convergence norm:
\[ |D| = \max D(X) + \max (d/dX)D(X) \]
(15)
with the maxima being taken over the domain of \( X \) if that set is compact. This metric is the Whitney \( C^1 \) metric. Both of these metrics have been used by many other economists in the differential topological approach to general equilibrium theory. See, for example, Mas-Colell (1985) for a survey. An important reason for using such metrics is their economic relevance. This is because any natural concept of closeness between functions should require their values to be close. The additional requirement that their first derivatives be close is sensible as these functions arise in solving either an optimization problem or an ordinary differential equation. Finally in the case of \( S \), second-order partial derivatives are required to be close in order to ensure that maximization as opposed to minimization occurs.
Define a map analogous to the excess demand function of a pure exchange economy, so that its zeros (in function space) are precisely the dividend signaling equilibria \((D(X), V_m(D))\). This is straightforwardly verified to be a continuous map, which implies the dividend signaling equilibrium correspondence is upper semi-continuous, with this result:

**THEOREM:** If \(W_n \to W\) in the Whitney \(C^2\) uniform-convergence metric and \(D_n(X)\) is the Pareto-undominated dividend signaling equilibrium function corresponding to \(W_n\), then \(D_n(X) \to D(X)\), in the Whitney \(C^1\) uniform-convergence metric, where \(D(X)\) is the Pareto-undominated dividend signaling equilibrium function corresponding to \(W\).

**PROOF:** The Pareto-undominated dividend signaling equilibrium functions \(D_n(X)\) must satisfy, the same boundary condition \(D_n(X_b) = D^*\) with \(X_b\) the bottom value current earnings take on, \(D^* = X_b - I^*\), and \(I^*\), the Fishierian optimum investment level with \(F'(I^*) = 1 + i\).

Let \(p\) denote the \(C^1\) uniform-convergence-norm-induced metric; \(D_1(X)\) denote the Pareto-undominated dividend signaling equilibrium that corresponds to the objective function

\[
W_1 = k_1V_m(D) + (1 - k_1)V^d(X, D)
\]

and \(D_2(X)\) denote the Pareto-undominated dividend signaling equilibrium corresponding to the objective function

\[
W_2 = k_2V_m(D) + (1 - k_2)V^d(X, D)
\]

Continuity of the mapping from \(S\) to the Pareto-undominated dividend signaling equilibrium implies:

**COROLLARY:** \(\forall \varepsilon > 0, \exists \delta > 0\) such that \(|k_1 - k_2| < \delta \Rightarrow p(D_1(X), D_2(X)) < \varepsilon\).

**PROOF:** This is simply a special case of the above theorem when the perturbations in \(W\) come from those in the fraction \(k\).

In a sense, this corollary extends the last sentence of Appendix B of Miller and Rock (1985), where a quantitative comparative statics result is proved, namely that increases in \(k\) lead to decreases in the slope of \(X(D)\) for \(F(I) = a \ln (1 + b)\) with \(a, b > 0\). From the first-order linear differential equation \(X'(D) = A - B(X - D)\), where the constant terms are given by

\[
A = (ka)^{-1}(a - (1 + i)b), \quad B = (1 + i)/ka
\]

and boundary condition \(X(D^*) = X_b\), the sign condition, \(\delta X'(D)/\delta k < 0\) can be derived. This is a quantitative comparative statics result that depends on the qualitative comparative statics result that small changes in \(k\) lead to small changes in \(X(D)\) and \(X'(D)\). This type of structural stability holds for any small deviation from \(W\). This includes minor perturbations in \(r\), the interest rate, or \(\gamma\), the persistence of earnings coefficient. Changes in \(k\), the fraction of the current stockholders who plan to sell their shares next period, can be interpreted as resulting from changes in the ownership structure of the firm.

**IV. CONCLUSIONS AND EXTENSIONS**

Structural stability of financial or accounting signaling models that satisfy assumptions A1–A6 of Riley (1979) has been proven. Continuity of the Pareto-undominated financial or accounting signaling equilibria under perturbations of the underlying basic environments defining these models extends their applicability to a larger class of environments, as described by firm objective and signaling cost functions. While this means the precise form of such functions is not as restrictive as might appear, there remains a stringent feature of all of these models, namely, their one-dimensional nature. In Ambarish, John, and Williams (1987), investment becomes another endogenous choice variable and signal in addition to dividends. But their model still only has a one-dimensional binomial quality variable, as does the model of Milgrom and Roberts (1986) of price and not directly informative advertisements as joint signals of new product quality by a monopolist. This industrial organization model is extended by Wilson (1985) to the case of a vector of possible signals, including dividends and charitable corporate contributions. Talmor (1981) attempted to construct a model in which both the mean and variance of a firm’s distribution of future cash flow could be signaled by dividend and investment choices. Hughes (1986) develops a bivariate signaling model of the market for initial public offerings, involving direct disclosure and retained ownership as joint signals of the mean and variance of the distribution of future cash flow of a new firm. Grinblatt and Hwang (1989) formulate a model that explains new issue underpricing involving both the offer price and the fraction of stock retained by the issuer as joint signals of the intrinsic value of the firm and the variance of its cash flows. In practice, any and all of the numerous financial or accounting decisions chosen by managers could be signals. This includes common stock repurchases, changes in the level of dividend payments, announcement of merger negotiations, and the method of payment used to finance leveraged buyouts or corporate takeovers. Daniel and Titman (1989) survey all of these “public money burning” signals. Brennan and Copeland (1988) models signaling by stock splits. But the development of a general multidimensional model incorporating all of these possible financial and accounting signals remains to be done. The structural stability of multidimensional signaling equilibria in such a canonical model should follow the method of proof in Huang (1987) for structural stability of multidimensional signaling equilibria in models that satisfy the assumptions of Quinzii and Rochet (1985) or in Huang (1989) for structural stability of multidimensional signaling equilibria in models that satisfy the assumptions of Engers (1987).
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NOTES

1. Non-dissipative signaling models are not considered in this paper. Ross (1977) suggests the choice between debt and equity finance can be a non-dissipative signal. Brennan and Kraus (1987) analyze costless financial signaling. A signal is non-dissipative if the information being signaled can be verified ex post. Non-dissipative signals are in essence contingent contracts, as noted by Spence (1976). Strong and Walker (1987) provide a clear distinction of this distinction.


3. Grundy (1989) has made similar points to those made here.

4. If the domain of W is not compact, then use the compact-open $C^2$ uniform-convergence norm, which replaces the domain of W with compact subsets of the domain of W.

5. If the domain of X is not compact, then use the compact-open $C^2$ uniform-convergence norm, which replaces the domain of X with compact subsets of the domain of X.

6. There is another desirable feature to using these norms, namely that S and P are Banach spaces. Using an equivalent norm, in the sense of inducing the same topology, would suffice.

7. Hughes and Schwartz (1968) and Hughes, Schwartz, and Thakor (1986) are accounting signaling models of the LIFO/FIFO choice.

REFERENCES


